

Parton branching and medium–induced radiation in a strongly coupled plasma

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Abstract

I review the parton picture at strong coupling emerging from the gauge/gravity duality together with its consequences for the energy loss and momentum broadening of a heavy quark moving through a strongly coupled plasma.

1. Motivation: Jet quenching at RHIC

Some of the experimental discoveries at RHIC, notably the unexpectedly large medium effects known as elliptic flow and jet quenching, led to the suggestion that the deconfined QCD matter produced in the intermediate stages of a heavy ion collision is strongly interacting. The phenomenon of jet quenching is particularly intriguing in that sense, as this probes the hadronic matter on a relatively hard scale (a few GeV), where QCD is *a priori* expected to be weakly coupled, by asymptotic freedom. One manifestation of this phenomenon is the ‘away jet suppression’ in Au+Au collisions: unlike in p+p or d+Au collisions, where the hard particles typically emerge from the collision region as pairs of back–to–back jets, in the Au+Au collisions one sees ‘mono–jet’ events in which the second jet is missing (at RHIC, ‘jet’ means leading particle). This has the following natural interpretation: the hard scattering producing the jets has occurred near the edge of the interaction region, so that one of the jets has escaped and triggered a detector, while the other one has been deflected, slowed down, or even absorbed, via interactions in the surrounding medium.

If this medium is composed of weakly interacting quasiparticles (quarks and gluons), then the deflection of the hard jet is due to its successive scattering off these quasiparticles, as illustrated in Fig. 1 left. This leads to the following estimate for the rate of transverse momentum broadening [1]

$$\hat{q} \equiv \frac{d\langle k_{\perp}^2 \rangle}{dt} \simeq \alpha_s N_c xG(x, Q^2), \quad (1)$$

where $xG(x, Q^2)$ is the gluon distribution in the medium on the resolution scale $Q^2 \sim \langle k_{\perp}^2 \rangle$ of the hard jet, as produced via quantum evolution from the medium intrinsic scale up to Q . For instance, if the medium is a finite–temperature plasma with temperature T , then $xG \simeq n_q(T) xG_q + n_g(T) xG_g$, where $n_{q,g}(T) \propto T^3$ are the quark and gluon densities in thermal equilibrium and $xG_{q,g}(x, Q^2)$ are the gluon distributions produced by a single quark, respectively gluon, on the scale $Q \gg T$. Some typical values at RHIC are $T \sim 0.4$ GeV and $Q \sim 2 \div 12$ GeV. At the LHC, they become $T \sim 0.7$ GeV and $Q \sim 100$ GeV (for actual jets). Assuming weak coupling, one can compute Eq. (1) in perturbation theory. But by doing that, one finds an estimate $\hat{q}_{\text{pQCD}} \simeq 0.5 \div 1$ GeV²/fm which is considerably smaller (by almost one order of magnitude) than the value extracted from the RHIC data ! This puzzle is comforted by the first data for Pb+Pb collisions at LHC, which confirm the strong jet quenching observed at RHIC and suggest that the medium effects can remain important even for jets as hard as $Q \sim 100$ GeV.

A possible solution to this puzzle is that the deconfined QCD matter is (relatively) strongly coupled. This would enhance the quantum evolution from T to Q and also the interactions between the hard probe and the medium. Note

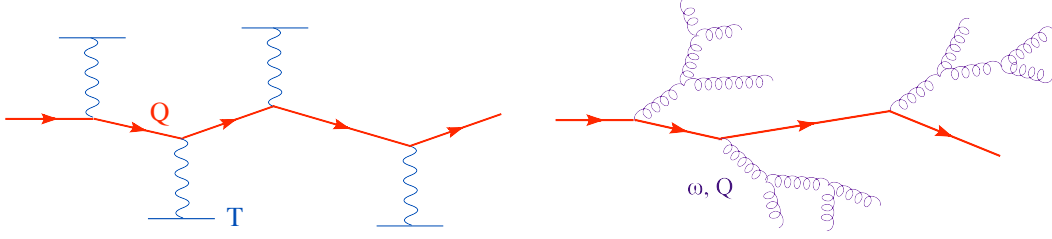


Figure 1: *Transverse momentum broadening for a heavy quark which propagates through a quark–gluon plasma. Left: weak coupling (successive scattering off medium constituents). Right: strong coupling (medium induced parton branching).*

that there is not necessarily a conflict with asymptotic freedom: to get an enhanced gluon distribution on the hard scale Q , it is enough to have a stronger coupling at the lower end of the evolution, that is, at the softer scale T . We have indeed $g(T) \sim 2$ for the temperatures T of interest at RHIC and LHC. Actually, it should be possible to study some aspects of this evolution in lattice QCD at finite- T and thus verify the hypothesis of strong coupling [2].

2. Parton picture and jet quenching at strong coupling

The previous discussion invites us to a better understanding of the properties of the quark–gluon plasma (QGP) at relatively strong coupling, that is, for $\alpha_s \equiv g^2/4\pi \sim 1$. However, even without the complications of confinement, the QCD calculations at strong coupling remain notoriously difficult. (In particular, lattice QCD cannot be used for real-time processes like transport phenomena and scattering.) So it has become common practice to look to the $\mathcal{N} = 4$ supersymmetric Yang–Mills (SYM) theory, whose strong coupling regime can be addressed within the AdS/CFT correspondence, for guidance as to general properties of strongly coupled plasmas.

$\mathcal{N} = 4$ SYM has the ‘color’ gauge symmetry $SU(N_c)$, so like QCD, but differs from the latter in some other aspects: it has (maximal) supersymmetry and hence conformal symmetry (the coupling g is fixed) and all the fields in its Lagrangian (gluons, scalars, and fermions) transform in the adjoint representation of $SU(N_c)$. But these differences are believed not to be essential for a study of the QGP phase of QCD in the temperature range of interest for heavy ion collisions at RHIC and LHC, that is, $2T_c \lesssim T \lesssim 5T_c$ with $T_c \simeq 170$ MeV the critical temperature for deconfinement. Indeed, lattice studies indicate that the QGP itself is nearly conformal in this window.

The AdS/CFT correspondence is the statement that the gauge theory $\mathcal{N} = 4$ SYM is ‘dual’ (*i.e.*, equivalent) to a special type of string theory living in the $D = 9 + 1$ space–time with $AdS_5 \times S^5$ geometry. This duality is particularly useful in that it relates the strong ‘t Hooft coupling limit $\lambda \equiv g^2 N_c \rightarrow \infty$ (with fixed $g \ll 1$) of the gauge theory to the weak–coupling limit of the string theory, in which the latter reduces to classical gravity (‘supergravity’ or SUGRA). The Anti-de-Sitter space–time AdS_5 can be viewed as the product between the physical space–time with $D = 3 + 1$ (the Minkowski boundary of AdS_5) and a fifth (or ‘radial’) dimension χ , with $0 \leq \chi < \infty$. Adding temperature on the gauge theory side corresponds to embedding a black hole (BH) in AdS_5 , with the BH horizon located at a distance $\chi = 1/T$ away from the Minkowski boundary at $\chi = 0$. (See Fig. 2 for some illustrations.)

Within this context, the gauge interactions between a hard probe and the strongly coupled plasma are described as the gravitational interactions between the dual bulk object (a string or a field) representing the hard probe and the BH. This amounts to solving the classical equation of motion for the propagation of the dual object in the AdS_5 BH space–time. For instance, one can unveil the parton structure of the plasma by studying ‘deep inelastic scattering’ (DIS), that is, the propagation of a virtual photon with space–like 4–momentum (see Fig. 2.a). The dual object in the bulk is a vector field A_m (with $m = \mu$ or χ) which obeys the Maxwell equations in curved space–time:

$$\partial_m (\sqrt{-g} g^{mp} g^{nq} F_{pq}) = 0, \quad \text{where} \quad F_{mn} = \partial_m A_n - \partial_n A_m. \quad (2)$$

The gravitational interactions are encoded in the 5D metric tensor g^{mn} , which involves the horizon of the BH.

Similarly, in order to study the phenomenon of ‘jet quenching’ for a heavy quark, one needs the corresponding ‘dual’ object in the bulk. This is a string hanging down in AdS_5 with one endpoint attached to the heavy quark on the boundary (see Fig. 2.b). The average string profile can be obtained by solving the corresponding equation of

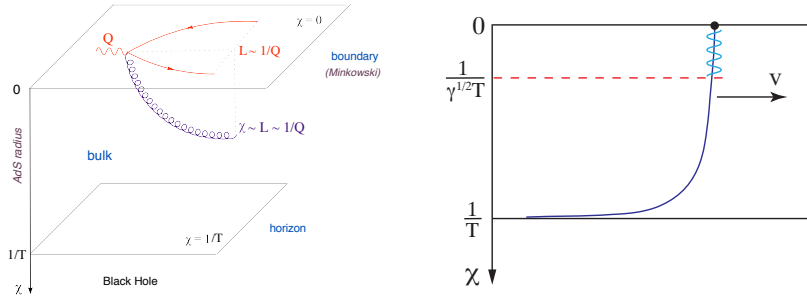


Figure 2: Left: dual description of a space-like current (the figure shows the associated bulk excitation and its ‘shadow’ on the boundary). Right: the trailing string dual to a heavy quark in the plasma (the dotted line marks the world-sheet horizon).

motion (the Nambu–Goto equation) in the background of the BH. Then the rate dE/dt for energy loss is computed as the flux of energy down the string. Also, the momentum broadening follows by studying the (small) fluctuations of the string along this average profile. These fluctuations reflect stochastic phenomena in the physical problem, whose precise nature will be discussed later on. This string problem is mathematically involved in the general case, but it has been fully carried out in one special situation, known as the ‘trailing string’ [4, 5]: this is the steady situation where the quark (and hence the string as a whole) moves at constant velocity under the action of an external force, which supplies the energy that is continuously lost towards the plasma/BH.

It turns out that the two problems alluded to above — DIS off the plasma and the quenching of a heavy quark — are connected at a deeper physical level: the partonic picture of the strongly-coupled plasma, as emerging from DIS [3], enables us to propose a physical interpretation for the AdS/CFT results for the trailing string, and also a rapid derivation of these results to parametric accuracy [6]. The key observation is that both phenomena are controlled by the same fundamental scale: a (space-like) virtuality scale that we shall refer to as the ‘saturation momentum’ Q_s . This scale is an intrinsic property of the medium (more precisely, of its parton distribution), although it also depends upon the energy of the ‘projectile’ (the hard probe) — the latter only fixes the energy resolution at which one probes the partonic distribution in the ‘target’ (the plasma).

Specifically, consider a space-like photon with 4-momentum $q^\mu = (\omega, 0, 0, q)$ and virtuality $Q^2 \equiv q^2 - \omega^2 \gg T^2$ which propagates through the plasma. By solving the AdS equation (2), one finds two physical regimes: (i) a *low energy* regime at $\omega \ll Q^3/T^2$, where there is essentially no interaction between the bulk field A_m and the BH (physically, this means that the plasma looks transparent to the virtual photon), and (ii) a *high energy* regime at $\omega \gtrsim Q^3/T^2$, where the field feels the attraction of the BH and eventually falls into it (*i.e.*, the photon is completely absorbed by the plasma). This strong energy dependence of the results is natural in the context of gravity, since gravitational interactions are proportional to the energy. The physical interpretation of these results back in the original gauge theory is more subtle and relies on the ‘UV/IR correspondence’. This correspondence states that the radial penetration χ of the bulk field in AdS_5 is proportional to the transverse size L of the partonic fluctuation of the virtual photon on the boundary (see Fig. 2.a). Consider *e.g.* a space-like photon in the vacuum: energy–momentum conservation implies that this photon cannot decay into on-shell quanta, but only fluctuate into a virtual partonic fluctuation with lifetime $\Delta t \sim \omega/Q^2$ and transverse size $L \sim 1/Q$, as determined by the uncertainty principle. And indeed, in the low energy regime above mentioned, the solution A_m to Eq. (2) is found to penetrate in AdS_5 (via diffusion: $\chi \sim \sqrt{t/\omega}$) up to a maximal distance $\chi \sim 1/Q$, which is reached after a time ω/Q^2 .

To similarly understand the high energy regime and the critical value $\omega \sim Q^3/T^2$ separating between the two regimes, one needs an additional piece of information: when viewing the equation of motion for the bulk field A_m through the prism of the UV/IR correspondence, one concludes that the strongly-coupled plasma acts on partonic fluctuations with a *tidal force* $F \sim T^2$ which pulls the partons apart. The partons become nearly on-shell when the mechanical work $W = F \Delta t$ provided by this force during the lifetime $\Delta t \sim \omega/Q^2$ of the fluctuation is large enough to compensate the energy deficit $\sim Q$ of the space-like system. This condition requires $T^2(\omega/Q^2) \gtrsim Q$ or $Q \lesssim Q_s(\omega, T)$, where $Q_s \sim (\omega T^2)^{1/3}$ is the plasma *saturation momentum*.

To summarize, a virtual photon with relatively high energy, or low virtuality $Q \lesssim Q_s(\omega, T)$, disappears into the

plasma because this photon can branch into partons under the action of the plasma force. Then the daughter partons can branch again and again, until the virtuality of the descendents along this partonic cascade is reduced to a small value of order T . When this happens, the fluctuation has *thermalized*: the partons become a part of the thermal bath.

A similar branching picture holds for the quanta emitted by a heavy quark which propagates with constant (average) velocity v through the plasma [6]. The virtual quanta with very high virtuality $Q \gg Q_s$ do not interact with the plasma and thus are reabsorbed by the quark: they are a part of its wavefunction. The low virtuality quanta with $Q \lesssim Q_s$ can decay under the action of the plasma force and thus initiate partonic cascades which eventually thermalize (cf. Fig. 1 right). As before, $Q_s \sim (\omega T^2)^{1/3}$ depends upon the energy ω of the emitted quanta, which in turn is constrained by $\omega/Q \lesssim \gamma$, with $\gamma = 1/\sqrt{1-v^2}$ (since the quanta cannot propagate faster than the heavy quark). The dissipation is controlled by the most energetic among the emitted quanta. By the previous arguments, these are the quanta with virtuality $Q \sim Q_s$ and with energy $\omega \sim Q_s \gamma$. The last condition together with $Q_s \sim (\omega T^2)^{1/3}$ imply $(Q_s)_{\max} = \sqrt{\gamma} T$. By also recalling that it takes a time $\Delta t \sim \omega/Q^2$ to emit a quanta with energy ω and virtuality Q , we conclude that the rate for energy loss can be estimated as follows:

$$-\frac{dE}{dt} \simeq \sqrt{\lambda} \frac{\omega}{(\omega/Q_s^2)} \simeq \sqrt{\lambda} Q_s^2 \sim \sqrt{\lambda} \gamma T^2. \quad (3)$$

The factor $\sqrt{\lambda}$ expresses the fact that, at strong coupling, the heavy quark radiates a large number of quanta, $\sim \sqrt{\lambda}$, in the time interval Δt .

Note that this mechanism for energy loss is different from the dominant respective mechanism as weak coupling. In both cases we can speak of medium-induced radiation. But the dynamics allowing for this radiation is different at weak coupling, where it involves medium rescattering (cf. Fig. 1 left), and respectively strong coupling, where it proceeds via medium-induced parton branching (cf. Fig. 1 right). In both cases, this mechanism also provides a transverse momentum broadening. At weak coupling, this has been estimated in Eq. (1). At strong coupling, this is generated by the random recoils associated with successive emissions, which are independent from each other. Each such an emission gives a maximal recoil $k_\perp \sim Q_s$ and their effects add in quadrature. This yields

$$\frac{d\langle k_\perp^2 \rangle}{dt} \sim \frac{\sqrt{\lambda} Q_s^2}{(\omega/Q_s^2)} \sim \sqrt{\lambda} \frac{Q_s^4}{\gamma Q_s} \sim \sqrt{\lambda} \sqrt{\gamma} T^3. \quad (4)$$

Eqs. (3) and (4) are consistent with the respective AdS/CFT results, as obtained from the trailing string [4, 5]. In these calculations, the saturation momentum $Q_s = \sqrt{\gamma} T$ appears as an induced horizon on the string world-sheet, at $\chi_h = 1/Q_s$. In that context, the string fluctuations are generated as ‘thermal noise’ at this induced horizon, with effective temperature $T = 1/z_h = Q_s$ [6]. This is similar to the Unruh effect in general relativity.

So far, we have assumed the medium to be ‘infinite’ (meaning much larger than the formation time $\Delta t \sim \omega/Q^2$ of a virtual quanta), but this is generally not the case in realistic heavy ion collisions. There a ‘hard probe’ like the heavy quark is produced as a *bare* parton via a hard scattering inside the medium. The time it takes that bare parton to dress itself with virtual quanta is generally larger (at least for the softer quanta) than the size L of the medium. Hence these quanta feel the effect of the plasma force $F \sim T^2$ over a time interval of order L , and not ω/Q^2 . Accordingly, the ‘saturation’ condition (mechanical work $\gtrsim Q$) becomes $Q \lesssim T^2 L \equiv Q_s$. With this new expression for Q_s , the previous estimates for the rate for energy loss and momentum broadening become [6]

$$-\frac{dE}{dt} \sim \sqrt{\lambda} Q_s^2 \sim \sqrt{\lambda} L^2 T^4, \quad \frac{d\langle k_\perp^2 \rangle}{dt} \sim \sqrt{\lambda} \frac{Q_s^2}{L} \sim \sqrt{\lambda} L T^4. \quad (5)$$

These equations imply $\Delta E \propto L^3 T^4$ and $\langle k_\perp^2 \rangle \propto L^2 T^4$, which show an enhanced medium dependence (by a factor $LT \gg 1$) as compared to the corresponding estimates at weak coupling [1].

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